Abelian Flavour Symmetries in Supersymmetric Models [1]

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Abstract

We propose a theory of flavour based on abelian horizontal gauge symmetries and modular invariances. We construct explicit supergravity models where the scale of the horizontal $U(1)$ symmetry breaking is fixed by the Green-Schwarz mechanism for anomaly cancellation. The supersymmetric spectrum is obtained in terms of the $U(1)$ charges which are determined by the Yukawa matrices.

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1 Introduction

There is a revival of interest in explaining the pattern of fermion masses and mixings by postulating a horizontal $U(1)_X$ gauge symmetry [1]-[5]. The $U(1)_X$ charges are assigned to fermions in such a way that only a small number of Yukawa interactions is allowed by the symmetry. The remaining effective Yukawa vertices are generated through non-renormalizable couplings to fields which are Standard Model gauge singlets but carry horizontal charge, in the effective theory defined at some large scale M_P (we shall identify it later with the Planck scale). When these singlet fields get vev's of $O(\varepsilon M_P), \varepsilon \leq 1$ and spontaneously break $U(1)_X$, the resulting Yukawa couplings are suppressed by powers of the small parameter, ε^{n_i}, where the powers n_i depend on the $U(1)_X$ charge assignment [6]. There are several reasons to assume this horizontal symmetry to be a local one. Of course, this avoids physical problems related to massless Goldstone bosons. Moreover, in the context of supersymmetric models with "stringy" $U(1)_X$ symmetry [7]-[8], this mechanism of fermion mass generation shows an interesting connection between phenomenologically viable mass pattern and the Green-Schwarz mechanism of anomaly cancellation, which successfully predicts the Weinberg angle [9]. It has also been suggested that the Fayet-Iliopoulos term that is fixed by the anomalies will naturally generate the small parameter ε in the $U(1)_X$ breaking.

In broken supergravity models with horizontal abelian symmetries the squark soft masses and the trilinear soft terms are correlated with the quark mass matrices by the symmetry. Generically, the off-diagonal entries (in the basis in which quarks are diagonal) are predicted [10] to be of the order of some powers of the Cabibbo mixing angle λ and in some cases comparable with the existing experimental limits. In a recent paper [11] it has been proposed to impose on such models the symmetries (and the spectrum) generic for effective supergravity lagrangian which originates from orbifold models of string compactifications. By combining the horizontal symmetry and the modular invariances that characterize these models, the soft terms can be calculated not only as powers of λ but also with their relative coefficients fixed in terms of the horizontal charges and modular weights.

If one consistently assumes that the hierarchies in the fermion mass spectrum are entirely due to the $U(1)_X$ symmetry, there are interesting relations between charges and modular weights. As a consequence, the sfermion (squarks and sleptons) spectrum is also predicted in terms of the $U(1)_X$ charges. Given a pattern of fermion mass matrices in terms of a set of

$U(1)_X$ charges, the sfermion masses are obtained in terms of two parameters characterizing the supersymmetry breaking, the gaugino mass M and the gravitino mass $m_{3/2}$. Now, this provides a unique way to test our horizontal symmetry models by a study of the sfermion mass spectrum, since the fermion masses and mixings are used to fix the charges and the $U(1)_X$ sector is in general too heavy to become visible.

By adopting the dilaton/moduli parametrisation of supersymmetry breaking proposed in [12] and in the case of one $U(1)_X$ symmetry spontaneously broken by the vev of a scalar field of charge $X = -1$, we get [11] (in the $U(1)_X$ basis)

$$
\begin{aligned}
\widetilde{m}_{q_i}^2 - \widetilde{m}_{q_j}^2 &= (q_i - q_j)\, m_{3/2}^2 \ , \\
\widetilde{m}_{q_i}^2 + \widetilde{m}_{u_j}^2 + \widetilde{m}_{h_2}^2 &= M^2 + (q_i + u_j + h_2)\, m_{3/2}^2 \ ,
\end{aligned}
\tag{1}
$$

where $\widetilde{m}_{q_i}$ ($\widetilde{m}_{u_i}$) is the diagonal element of the sfermion mass matrix and q_i (u_i) the $U(1)_X$ charge associated with the left-handed (right-handed up) quark of the ith family, and $\widetilde{m}_{h_2}$ and h_2 are the corresponding quantities for one of the Higgs doublets. Regarding the trilinear soft terms we get, e.g., the prediction

$$
A_{ij}^U \simeq -M + (q_i + u_j + h_2)\, m_{3/2}
\tag{2}
$$

for the coefficient of trilinear coupling $Y_{ij}^U Q^i U^j H_2$. The complete relations exhibit flavour off-diagonal terms in the soft masses and the full trilinear soft terms are not proportional to the corresponding Yukawa couplings. As will be explained, these additional contributions give physical observable effects of the same order as the "diagonal" ones displayed in (1) and (2). Similar relations hold for down-type squarks and sleptons. It must be noticed that the simple results (1) - (2) follow from a cancellation between the geometrical supergravity contribution and the part of the mass splitting from the $U(1)_X$ D-term that contain model dependent parameters. The horizontal splitting in (1) is proportional to the charge differences which appear in the Yukawa mass matrices. Since the lighter fermions are associated to larger charges, the corresponding sfermions are predicted to be heavier than superpartners of heavier fermions.

Extensions of the Standard Model of the electroweak interactions are constrained by flavour changing neutral current (FCNC) processes like $K^0 - \bar{K}^0$ mixing, $b \to s\gamma$, $\mu \to e\gamma$ or the electric dipole moment (e.d.m.) of the neutron. The observed suppression of FCNC transitions is nicely explained

in the Standard Model (SM) by the GIM mechanism. The supersymmetric extensions of the SM do contain additional contributions to FCNC transitions from sfermion exchange in loop diagrams. They can be potentially dangerous if, in the basis in which fermion mass matrices are diagonal, the sfermion mass matrices have large flavour off-diagonal entries. In addition, new phases which are usually present in the sfermion mass matrices and in the trilinear couplings can give too large CP violating effects (e.g. too large neutron e.d.m.). After a rotation to the quark diagonal basis, we obtain our prediction for the squark mass matrices up to an overall scale. The question one may ask in our class of models is this: can one suppress the off-diagonal entries in the squark mass matrices below the order of magnitude estimate based on the $U(1)_X$ symmetry alone? A priori, this might occur if some coefficients vanish, i.e. for certain choice of horizontal charges. The conclusion of ref.[11] is that such an additional off-diagonal suppression does not hold for the phenomenologically acceptable quark masses. In the case of one $U(1)_X$ symmetry, squarks masses can be neither universal nor aligned with quark masses.Thus, the models of this class predict some deviations from the Standard Model predictions for the FCNC transitions to be eventually observed at higher level than in the case of universal soft masses at the Planck scale.

In the next three Sections of this paper we discuss the case of one $U(1)_X$ symmetry. We extend the formalism of ref.[11] by allowing for flavour dependent modular weights for the superpotential and give some details of the calculation of the soft terms. The breaking of the horizontal $U(1)_X$ is investigated and the induced D-term is evaluated. In Section 5 , the Higgs sector mass parameters are obtained in terms of their $U(1)_X$ charges, which are restricted by the fermion masses. Then, it is shown that the requirement of proper electroweak symmetry breaking strongly constrains the scales $m_{3/2}$ and M. In consequence, the magnitude of the FCNC effects is also determined. It is interesting to notice that this highly predictive class of models with one $U(1)_X$ symmetry, although only marginally acceptable from the point of view of FCNC effects, is qualitatively consistent with the existing phenomenology and, moreover, gives testable predictions for superpartners masses.

In Section 6 we consider a class of models with two abelian horizontal symmetries. They have been suggested [10] to reduce flavour non-diagonal entries in the scalar mass matrices and so to alleviate FCNC effects. By an ad hoc choice of charges, one can further suppress $\bar{K}K$ and $\bar{D}D$ mixings. The relation between the modular weights and horizontal charges are derived for

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this case and used to calculate the soft mass terms. It becomes clear from our results that, with any number of abelian symmetries, it is impossible to get either universal or aligned (with quarks) squark masses for acceptable quark mass matrices and therefore the models predict interesting phenomenology in the FCNC sector.

In the last section we summarize the main features of the models with horizontal $U(1)'s$ and modular invariances from the phenomenological point of view.

2 Yukawa matrices from horizontal $U(1)_X$ gauge symmetry

A natural way of understanding the fermion mass matrices is to postulate a family (horizontal) gauge symmetry spontaneously broken by the vacuum expectation values (vev's) of some scalar fields ϕ which are singlets under the Standard Model gauge group. The hierarchy of fermion masses and mixing angles is then explained by assigning different charges to different fermions. Only the third family of fermions acquires a mass at the tree level and all the other Yukawa couplings are forbidden by the $U(1)_X$ symmetry. After spontaneous symmetry breaking of the $U(1)_X$ symmetry, higher order invariant terms in the lagrangian (or superpotential in the supersymmetric case) can be written and have the form $(\frac{<\phi>}{M_P})^{n_{ij}} \bar{\psi}_i \psi_j H$ (after decoupling of the heavy fields), where ψ_i are the SM fermions, H is a Higgs field and M_P is a large scale. Postulating $\varepsilon \equiv \frac{\langle\phi\rangle}{M_P} \simeq \lambda$ (the Cabibbo angle) one can easily explain hierarchies in the effective Yukawa couplings, precisely in the simplest case of abelian symmetry, with all the coefficients of the higher dimension operators of the order $O(1)$.

In the context of the string inspired models, the most natural candidate for such a symmetry is the anomalous $U(1)_X$ gauge group present in most of the known 4-dimensional string models [7, 8]. In this case, one-loop triangle graphs generate Adler-Bell-Jackiw gauge anomalies. The gauge symmetry is restored by the 4-dimensional version of the Green- Schwarz mechanism [13], by the use of the dilaton-axion superfield. The relevant terms in the Lagrangian are

$$\mathcal{L}_{gauge} = \frac{1}{4} \sum_a k_a S \int d^2\theta (\ tr W^\alpha W_\alpha)_a \ , \qquad (3)$$

$$K = -ln(S + S^+ + \delta_{GS}V) + ... \ .$$

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In (3), V and W_α are respectively the $U(1)_X$ gauge superfield and the gauge superfield strenght, S is the dilaton superfield, k_a is the Kac-Moody level of the factor G_a of the gauge group $G = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_X$, and δ_{GS} is the Green-Schwarz coefficient. Under a $U(1)_X$ gauge transformation, S is shifted as $S \rightarrow S + i\delta_{GS}\alpha(x)$. The complete Lagrangian is gauge invariant provided the anomaly coefficients A_i satisfy the condition

$$\delta_{GS} = \frac{A_1}{k_1} = \frac{A_2}{k_2} = \frac{A_3}{k_3} = \frac{A_X}{k_X} \ , \tag{4}$$

where δ_{GS} is computed to be $\delta_{GS} = \frac{1}{192\pi^2} \, TrX$. The mixing $S - V$ in the Kähler potential gives rise to a term $(g_X = 1/(S + S^\dagger))$

$$V_D = \frac{g_X^2}{2} \left(\sum_A K_A X_A \phi^A + \frac{M_P^2}{192\pi^2} \, TrX \right)^2 \ , \tag{5}$$

in the scalar potential, where A labels all chiral fields of charges X_A and $K_A = \frac{\partial K}{\partial \phi^A}$. The last term in the parenthesis in (5) is the coefficient of the Fayet-Iliopoulos D-term [14], which forces at least one of the fields to get a vacuum expectation value and to break the $U(1)_X$ gauge symmetry at a scale slightly below the Planck scale, depending on the value of TrX. If $n > 1$ $U(1)$ symmetries are considered, we can always define $n - 1$ anomaly free symmetries and the present discussion applies. Recently, it has been shown [1] that using this symmetry and imposing phenomenologically successful fermion mass matrices one correctly predicts the Weinberg angle at the scale where $U(1)_X$ is spontaneously broken. This indicates a close relation between the Green-Schwarz mechanism and $U(1)_X$ fermion mass matrices. Also, a direct connection between fermion mass matrices and mixed gauge anomalies was established [2] and further studied in [4, 5]. Within this scheme the scale M_P of the effective theory is assumed to be the Planck mass, while the parameter ε^2 is equal to $-(M_P^2 TrX)/(192\pi^2 X_\phi)$.

Let us now consider the case when one SM gauge singlet takes a vev and all other matter field vev's are zero at the Planck scale. Supersymmetry is assumed, so that the Yukawa couplings are encoded in the $U(1)_X$ invariant superpotential

$$
\begin{aligned}
W \ = \ & \sum_{ij} \left[Y_{ij}^U \, \theta \left(q_i + u_j + h_2 \right) \left(\frac{\phi}{M_P} \right)^{q_i + u_j + h_2} Q^i U^j H_2 \right. \\
& \left. + Y_{ij}^D \, \theta \left(q_i + d_j + h_1 \right) \left(\frac{\phi}{M_P} \right)^{q_i + d_j + h_1} Q^i D^i H_1 \right.
\end{aligned}
$$

$$+ Y_{ij}^E \, \theta \, (\ell_i + e_j + h_1) \left(\frac{\phi}{M_P} \right)^{\ell_i + e_j + h_1} L^i E^j H_1 \Bigg] , \qquad (6)$$

where we denote the matter fields by capitals Φ^i, the corresponding $U(1)_X$ charges by small letters φ_i (after choosing the normalization of the $U(1)_X$ to be such that the singlet charge is $X_\phi = -1$.) The θ-functions remove the Yukawa couplings that are forbidden by the $U(1)_X$ symmetry combined with the holomorphicity of W. We also assume R-parity symmetry in W (see [15] for attempts to enforce R-parity through horizontal symmetries). All the allowed entries in the Yukawa coupling matrices Y^D, Y^D, Y^E are assumed to be "natural", of $O(1)$. The scalar potential in (5) vanishes for

$$\phi^2 = \epsilon^2 M_P^2 = \frac{TrX}{192\pi^2} M_P^2 \ , \qquad (7)$$

if we postulate $TrX > 0$. As stated before we assume ϵ to be of the order of the Cabibbo angle. By a choice of the $U(1)_X$ charges we get the powers of ϵ in the effective low energy Yukawa couplings,

$$\hat{Y}_{ij}^U = Y_{ij}^U \epsilon^{(q_i + u_j + h_2)} \theta(q_i + u_j + h_2) \qquad (8)$$

(analogously for $\hat{Y}^D$, $\hat{Y}^E$) needed to implement their hierarchy.

Before proceeding to study the consequences of this broken horizontal symmetry in the supergravity framework, let us pay some attention to the question of the uniqueness of the solution (7). Indeed, in the supersymmetric limit one expects degenerate minima of the scalar potential. They correspond to the vanishing of all the auxiliary fields: the gradients of the superpotential (F-type) and the D-terms associated to each one of the $SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \bigotimes U(1)_X$ gauged symmetries. It has been shown that the D-terms vanish at and only at those values of the field that correspond to extrema of holomorphic invariant polynomials [16] (with the exception of D_X-terms that contain a Fayet-Iliopoulos constant) . This allows for a systematic classification of the zeros of the $SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y$ D-terms. For instance, the invariant $Q^i D^j H_1$ corresponds to a direction $Q^i = D^j = H_1 = v$ in the field space where all these D-terms vanish, while $D_X = 0$ may be written as[3]

$$g_X \left(-|\phi|^2 + (q_i + d_j + h_1)v^2 + \epsilon^2 M_P^2 \right) = 0 \ . \qquad (9)$$

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[3]The vev's considered here are to be understood in the canonical basis as discussed in the next sections.

Combining similar solutions corresponding to all other invariants, one defines a manifold of solutions of D-terms. Those that corespond to non-vanishing gradients of the superpotential W break supersymmetry and are not minima of the scalar potential (at large scales). So, in our example (9), one easily checks from (6) that for $v \neq 0$ and $\phi \neq 0$, the scalar potential will vanish only if all the entries in the ith row and jth column of the matrix Y^D vanish. This is obviously excluded in phenomenologically viable models. Of course the same arguments applies for solutions associated to the Y^U and Y^E terms in (6). The solutions of (9) with $\phi = 0$ are also excluded if $q_i+d_j+h_1 \geq 0$, $q_i+u_j+h_2 \geq 0$ and $l_i+e_j+h_1 \geq 0$. Therefore these conditions are sufficient (though they seem also necessary at this stage, they can be avoided as discussed herebelow) to ensure the uniqueness of the solution (7) if R-parity is assumed. Next, one has to take care of solutions coming from R-parity violating invariants such as $L^k Q^i D^j$, where the Higgs H_1 is replaced in (9) by one of the sleptons L^k, and $L^k = Q^i = D^j \neq 0$. For $\phi \neq 0$, the scalar potential gets a non-vanishing contribution if $Y_{ij}^D \neq 0$, ie if $q_i + d_j + h_1 \geq 0$. Analogously, from $L^k L^i E^j$ invariants one deduces the condition $h_1 + l_i + e_j \geq 0$. Since the experimental data allow for some violation of R-parity, one can replace the positivity condition above by a condition on the non-vanishing of rows and columns of the R-parity violating Yukawa couplings. In this case, the solutions of (9) with $\phi = 0$, when allowed, will be degenerate with that in (7). They correspond, however, to very isolated points in the field space and (7) should remain stable, if not unique. As mentioned earlier, the positivity conditions are sufficient but not necessary. Other invariants with more than three matter and Higgs superfields can be included in the superpotential without any consequence on the Froggat-Nielsen mechanism. They may help to avoid the supersymmetric minima. For instance, a (R-parity conserving) term like $(H_1 Q^i D^j)^2$ in W would allow for a zero in the Y^D matrix with $h_1 + q_i + d_j < 0$. Notice that the (sufficient) positivity conditions are consistent with most of the models considered in the literature [1]-[5].

Finally, let us consider the two bilinear invariants $H_1 H_2$ and $L^k H_2$. The vanishing of D-terms leads to $H_1 = H_2 = v$ and $-\phi^2 + (h_1 + h_2)v^2 + \epsilon^2 M_P^2 = 0$, for the former, and similarly for the latter with $H_1 \rightarrow L^k$ (the combination of these solutions are, in general, forbidden by non-zero entries in Y^E). However, at this level, one cannot argue that the degeneracy is lifted by the term $\mu H_1 H_2$ in the superpotential, since it has to be absent in the supersymmetric limit, even if $(h_1 + h_2) > 0$. For phenomenological

reasons, it has to be related to the supersymmetry breaking scale (the so-called $\mu-$problem). We postpone the question of this particular degeneracy until section 4.

3 $U(1)_X$ and modular symmetries in effective supergravity from string models

The phenomenologically interesting low energy limit of the superstring models is the $N = 1$ supergravity defined by the Kähler function K, the superpotential W and the gauge kinetic function f. The fields in the massless string spectrum are a universal dilaton S, moduli fields with no scalar potential generically denoted by T_α and matter chiral fields Φ^i which include the SM particles, $\Phi^i = Q^i, U^i, D^i, L^i, E^i, H_1, H_2$. An important role in the following discussion will be played by the target-space modular symmetries $SL(2, Z)$ [17] associated with the moduli fields $T_\alpha(\alpha = 1..p)$, acting as $T_\alpha \to (a_\alpha T_\alpha - ib_\alpha)/(ic_\alpha T_\alpha + d_\alpha)$, with $(a_\alpha d_\alpha - b_\alpha c_\alpha) = 1$ and $a_\alpha...d_\alpha \in Z$. In effective string theories of the orbifold type, the matter fields Φ^i transform under $SL(2, Z)$ as $\Phi^i \to (ic_\alpha T_\alpha + d_\alpha)^{n_i^{(\alpha)}} \Phi^i$, where the $n_i^{(\alpha)}$ are called the modular weights of the fields Φ^i with respect to the modulus T_α [18]. These modular transformations, which are symmetries of the supergravity theory, can be viewed as a particular type of Kähler transformations.

We assume the existence of superstring models which in the low-energy limit yield effective supergravity theories with the above minimal content of superfields, the gauge group $SU(3)_C \bigotimes SU(2)_L \bigotimes U(1)_Y \bigotimes U(1)_X$ and a SM singlet supermultiplet ϕ with $X_\phi = -1$. The superpotential W is defined in (6) and the Kähler potential K is as follows,

$$
\begin{aligned}
K & = K_0\left(T_\alpha, \bar{T}^{\bar{\alpha}}\right) - \ln\left(S + \bar{S}\right) + \prod_{\alpha=1}^{p} t_\alpha^{n_\phi^{(\alpha)}} \bar{\phi}\phi \\
& + \sum_{\Phi^i = Q^i, U^i, D^i, L^i, E^i, H_1, H_2} K_{i\bar{j}}^{\Phi} \Phi^i \bar{\Phi}^{\bar{j}},
\end{aligned}
$$

$$
K_{i\bar{j}}^{\Phi} = \delta_{ij} \prod_{\alpha=1}^{p} t_\alpha^{n_i^{(\alpha)}} + Z_{i\bar{j}}^{\Phi}\left[\theta\left(\varphi_i - \varphi_j\right) \prod_{\alpha=1}^{p} t_\alpha^{n_j^{(\alpha)} + \bar{n}_{\Phi, i\bar{j}}^{(\alpha)}} \left(\frac{\phi}{M_P}\right)^{\varphi_i - \varphi_j} + \right.
$$

$$
\left. \theta\left(\varphi_j - \varphi_i\right) \prod_{\alpha=1}^{p} t_\alpha^{n_i^{(\alpha)} + n_{\Phi, i\bar{j}}^{(\alpha)}} \left(\frac{\bar{\phi}}{M_P}\right)^{\varphi_j - \varphi_i} \right] + ... \tag{10}
$$

where $i, j = 1, 2, 3$. In (10), t_α are the real part of the p moduli fields T_α and

the dots stand for higher order terms in the fields ϕ and Φ^i. Note that the flavour non-diagonal terms in the Kähler potential, proportional to the coefficients $Z_{i\bar{j}}^{\Phi}$, are constrained only by the gauge symmetry and R-parity and have the form $K_{i\bar{j}}^{Q} Q^i \bar{Q}^{\bar{j}}$, $K_{i\bar{j}}^{U} U^i \bar{U}^{\bar{j}}$, $K_{i\bar{j}}^{D} D^i \bar{D}^{\bar{j}}$, $K_{i\bar{j}}^{L} L^i \bar{L}^{\bar{j}}$ and $K_{i\bar{j}}^{E} E^i \bar{E}^{\bar{j}}$. The explicit dependence on t_α in these terms is fixed by the modular invariance conditions, which are discussed in detail below. In general, the coefficients $Z_{i\bar{j}}^{\Phi}$ are automorphic functions of the moduli, of chiral weight $n_{\Phi,i\bar{j}}^{(\alpha)}$ and antichiral weight $\bar{n}_{\Phi,i\bar{j}}^{(\alpha)}$, i.e. they transform under the modular transformations as $Z_{i\bar{j}}^{\Phi} \to (ic_\alpha T_\alpha + d_\alpha)^{n_{\phi,i\bar{j}}^{\alpha}} (-ic_\alpha T_\alpha^+ + d_\alpha)^{\bar{n}_{\phi,i\bar{j}}^{\alpha}} Z_{i\bar{j}}^{\Phi}$. The coefficients $Y_{ij}^{U}, Y_{ij}^{D}, Y_{ij}^{E}$ in (6) can also be automorphic functions of the moduli fields, of weight $n_{U,ij}^{(\alpha)}$, etc. Note that a coefficient with two analytic indices, like $n_{U,ij}^{(\alpha)}$, is related to the modular transformations of a Yukawa coefficient, here Y_{ij}^{U}, while a coefficient with an analytic and an antianalytic indices, like $n_{U,i\bar{j}}^{(\alpha)}$, is related to the modular transformations of a non-diagonal Kähler coefficient, here $K_{i\bar{j}}^{U}$.

In order to impose the modular symmetries, let us first define $n_0^{(\alpha)}$ by the modular transformations of the Kähler potential for the moduli fields, $K_0 \to K_0 + n_0^{(\alpha)} \ln |ic_\alpha T_\alpha + d_\alpha|^2$, which is a Kähler transformation. A typical example is

$$K_0 \;=\; -\sum_{\alpha=1}^{p} n_0^{(\alpha)} \ln \, t_\alpha. \tag{11}$$

The modular invariance of the full Kähler potential requires the following relations between modular weights and $U(1)_X$ charges

$$(\varphi_i - \varphi_j) \, n_\phi^{(\alpha)} \;=\; X_\phi \left(n_i^{(\alpha)} - n_j^{(\alpha)} + n_{\Phi,i\bar{j}}^{(\alpha)} - \bar{n}_{\Phi,i\bar{j}}^{(\alpha)} \right) , \tag{12}$$

where $i, j = 1, 2, 3$ are family indices, X_ϕ is the $U(1)_X$ charge of the singlet ϕ and φ_i are $U(1)_X$ charges for fermions with the same SM quantum numbers. So (12) is a horizontal (family) relation applying separately for Q, U, D, L and E fermions.

From the superpotential, to be consistent with modular invariance of the complete theory, we get the following conditions for the quarks and leptons:

$$-X_\phi^{-1} (q_i + u_j + h_2) \, n_\phi^{(\alpha)} + n_{q_i}^{(\alpha)} + n_{u_j}^{(\alpha)} + n_{h_2}^{(\alpha)} + n_0^{(\alpha)} + n_{U,ij}^{(\alpha)} \;=\; 0,$$

$$-X_\phi^{-1} (q_i + d_j + h_1) \, n_\phi^{(\alpha)} + n_{q_i}^{(\alpha)} + n_{d_j}^{(\alpha)} + n_{h_1}^{(\alpha)} + n_0^{(\alpha)} + n_{D,ij}^{(\alpha)} \;=\; 0,$$

$$-X_\phi^{-1} (l_i + e_j + h_1) \, n_\phi^{(\alpha)} + n_{l_i}^{(\alpha)} + n_{e_j}^{(\alpha)} + n_{h_1}^{(\alpha)} + n_0^{(\alpha)} + n_{E,ij}^{(\alpha)} \;=\; 0. \tag{13}$$

Each of these equations must hold if the corresponding Yukawa term is not zero. From (13), we get the relation

$$(q_i - q_j) n_\phi^{(\alpha)} = X_\phi \left(n_{q_i}^{(\alpha)} - n_{q_j}^{(\alpha)} + n_{U,ik}^{(\alpha)} - n_{U,jk}^{(\alpha)} \right), \qquad (14)$$

as a consequence of the existence of the Yukawa couplings Y_{ik}^U and Y_{jk}^U in the superpotential. Similar relations are obtained by replacing q_i by u_i, d_i, l_i, e_i. Comparing (14) with (12), we get additional conditions between the modular transformations of the off-diagonal terms in the Kähler potential and the transformations of the Yukawa couplings, namely,

$$n_{U,ik}^{(\alpha)} - n_{U,jk}^{(\alpha)} = n_{D,ik}^{(\alpha)} - n_{D,jk}^{(\alpha)} = n_{Q,i\bar{j}}^{(\alpha)} - \bar{n}_{Q,i\bar{j}}^{(\alpha)} \equiv s_{Q,ij}^{(\alpha)} \ ,$$
$$n_{U,ki}^{(\alpha)} - n_{U,kj}^{(\alpha)} = n_{U,i\bar{j}}^{(\alpha)} - \bar{n}_{U,i\bar{j}}^{(\alpha)} \equiv s_{U,ij}^{(\alpha)} \ ,$$
$$n_{D,ki}^{(\alpha)} - n_{D,kj}^{(\alpha)} = n_{D,i\bar{j}}^{(\alpha)} - \bar{n}_{D,i\bar{j}}^{(\alpha)} \equiv s_{D,ij}^{(\alpha)} \ ,$$
$$n_{E,ki}^{(\alpha)} - n_{E,kj}^{(\alpha)} = n_{E,i\bar{j}}^{(\alpha)} - \bar{n}_{E,i\bar{j}}^{(\alpha)} \equiv s_{E,ij}^{(\alpha)} \ ,$$
$$n_{E,ik}^{(\alpha)} - n_{E,jk}^{(\alpha)} = n_{L,i\bar{j}}^{(\alpha)} - \bar{n}_{L,i\bar{j}}^{(\alpha)} \equiv s_{L,ij}^{(\alpha)} \ , \qquad (15)$$

which are all independent of the values of $k = 1, 2, 3$. Actually, by putting the $U(1)_X$ charges to zero in (12) and (13) one finds that the relations (15) are independent of the $U(1)_X$ symmetry and apply more generally. They restrict the moduli dependence of the Yukawa couplings. Fourteen of these equations are independent, ten in the quark sector and four in the leptonic sector. So, if the quark Yukawa couplings are a priori defined by eigthteen modular weights, one for each coupling, only eight of them are independent. Analogously, the lepton Yukawa couplings are defined by five independent modular weights.

A simple and interesting case is when the Yukawa couplings do not depend on the moduli fields or the dependence is flavour blind and Y are of $O(1)$. We shall mainly consider this case, to be consistent with our physical assumption in this paper, that the fermion mass hierarchy is entirely due to the $U(1)_X$ charges. Then the relations (15) also imply $n_{\Phi,i\bar{j}}^{(\alpha)} = \bar{n}_{\Phi,i\bar{j}}^{(\alpha)}$, etc., and a real type modular transformations $Z_{i\bar{j}}^\Phi \to |ic_\alpha T_\alpha + d_\alpha|^{2n_{\Phi,i\bar{j}}^{(\alpha)}} Z_{i\bar{j}}^\Phi$. In this case (14) simplifies to

$$(\varphi_i - \varphi_j) n_\phi^{(\alpha)} = X_\phi \left(n_i^{(\alpha)} - n_j^{(\alpha)} \right). \qquad (16)$$

The relations (16) give a connection between the modular weights and the $U(1)_X$ charges and can be interpreted as an embedding of the $U(1)_X$ symmetry in the modular symmetries. They enable us to write the Kähler metric

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for the matter fields as

$$K_{i\bar{j}}^{\Phi} \;=\; \prod_{\alpha=1}^{p} t_{\alpha}^{(n_i^{(\alpha)}+n_j^{(\alpha)})/2} \left(\delta_{i\bar{j}} + Z_{i\bar{j}}^{\Phi} \hat{\varepsilon}^{|\varphi_i - \varphi_j|} \right) , \qquad (17)$$

where the small parameter $\hat{\varepsilon} = \prod_{\alpha} t_{\alpha}^{n_{\phi}^{(\alpha)}/2} \frac{<\phi>}{M_P}$ settles the hierarchy in the fermion and scalar mass matrix elements (if, for some $(i\bar{j})$, (16) is not fulfilled, the coefficient vanishes).

If eq.(14) are not satisfied, modular invariance of the superpotential implies zeroes in the Yukawa matrices and in the off-diagonal entries of the Kähler metric. These type of zeroes must be distinguished from the ones given by $U(1)_X$ invariance and the holomorphicity of the superpotential W as described by the θ-functions in (6). We could try to construct phenomenologically interesting models in this way, in the spirit of ref.[19]. However, as argued in [11], due to the fact that modular symmetry zeroes in Yukawa matrices imply zeroes in the corresponding off-diagonal elements of the Kähler metrics, the zero textures of the above matrices are preserved in the fermion canonical basis. Phenomenologically, they can acommodate the fermion masses and one mixing angle, but they cannot explain the whole V_{CKM} matrix. Hence, for the quarks, the relations (14) must be imposed for *all* the indices (i,j) .

The physical Yukawa couplings $\hat{Y}$ are obtained by the canonical normalization of the kinetic terms, which requires the redefinition of the fields $\hat{\Phi}^i = e_j^i \Phi^j$ where the vielbein $e_j^i(t_\alpha, \phi)$ verifies

$$K_{i\bar{j}}^{\Phi} \;=\; \delta_{\bar{k}l} \bar{e}_{\bar{j}}^{\bar{k}} e_i^l . \qquad (18)$$

The potential effect of these field redefinitions is to remove the eventual zeroes in the Yukawa matrices coming from the holomorphicity and $U(1)_X$ invariance of the superpotential (Examples of this type of a phenomenological interest can be found in [4]).

4   Predictions for the soft terms

The spontaneous breaking of local supersymmetry gives rise to a low-energy global supersymmetric theory together with terms that explicitly break supersymmetry, but in a soft way. The signal of supersymmetry breaking is in non zero vev's of the auxiliary components of the chiral superfields $F^a = e^{\frac{G}{2}} G^a$, where $G^a = K^{a\bar{b}} \partial_{\bar{b}} G$ and $G = K + \ln |W|^2$. We consider only the

case of zero tree level cosmological constant, i.e., we impose $< G^A G_A >= 3$ and the order parameter for the supergravity breaking is provided by the gravitino mass $m_{3/2}^2 = e^G$. A complete scenario of supersymmetry breaking is still missing. A pragmatic attitude was taken in [12], where a parametrization of the supersymmetry breaking was proposed, quite independent of its specific mechanism. The fields which participate in the supergravity breaking were assumed to be the moduli T_α and the dilaton S. The parametrization is[4]

$$
\begin{aligned}
G^\beta &= \sqrt{3}\Theta_\beta t_\beta, \\
G^\beta G_\beta &= 3\cos^2\theta, \\
G^S G_S &= 3\sin^2\theta.
\end{aligned}
\tag{19}
$$

The angle θ and the Θ_α's parametrize the direction of the goldstino in the T_α, S space. The normalization of the Θ_α is fixed by (19). If (11) is assumed we get $\sum_\alpha n_0^{(\alpha)}\Theta_\alpha^2 = \cos^2\theta$. In the presence of the $U(1)_X$ symmetry spontaneously broken close to the Planck scale there is an additional contribution to supersymmetry breaking with $< G^\phi G_\phi >\sim < \phi >^2$. More generally, any field with a large vev (non negligible compared to M_P) contributes to supersymmetry breaking.

The soft terms are computed from the usual expressions of supergravity, but with the flavour non-diagonal Kähler potential, eq.(10). It is worth noticing that only the lowest power of $\hat{\varepsilon}$ or ϕ have been included in (10). Therefore, the predictions herebelow have also been derived to the lowest power of $\hat{\varepsilon}$. In this approximation, it is straightforward to find

$$
G^\phi = (1 - \sqrt{3}n_\phi^{(\alpha)}\Theta_\alpha)\phi .
\tag{20}
$$

Even with $G^\phi \neq 0$ in eq. (20), the parametrization (19) is consistent with the vanishing of the cosmological constant in the leading order in $\hat{\varepsilon}$.

The soft terms are computed from the scalar potential, which in our case reads ($a = T^\alpha, S, \phi$, matter fields).

$$
V = e^G \left(G_A(G^{-1})^{A\bar{B}}G_{\bar{B}} - 3\right) + \frac{g_X^2}{2}\left(K^A\varphi_A\phi_A + \delta_{GS}M_P^2\right)^2.
\tag{21}
$$

Since the soft parameters are relevant for low-energy phenomenology, it is more appropriate to express them *after the field redefinition* that brings the kinetic terms to their canonical forms as consistently done in the following.

---

[4]Our definition of Θ_α in (19) is different from that in [12].

Let us first consider the soft scalar masses that have the standard supergravity expression [20]

$$
\begin{aligned}
\widetilde{m}_{i\bar{\jmath}}^2 &= (\widetilde{m}_{i\bar{\jmath}}^2)_F + (\widetilde{m}_{i\bar{\jmath}}^2)_D \\
&= \left(G_{i\bar{\jmath}} - G^\alpha \hat{R}_{i\bar{\jmath}\alpha\bar{\beta}} G^{\bar{\beta}} \right) m_{3/2}^2 + g_X \min(\varphi_i, \varphi_j) G_{i\bar{\jmath}} < D >, \quad (22)
\end{aligned}
$$

where $\hat{R}_{i\bar{\jmath}\alpha\bar{\beta}}$ is the Riemann tensor of the Kähler space and $< D >$ stands for the contribution from the D-term of the $U(1)_X$ gauge group,

$$
\begin{aligned}
D_X &= g_X(K_A \varphi_A \phi^A + \delta_{GS} M_P^2) \\
&= g_X \left(- \prod_{\alpha=1}^{p} t_\alpha^{n_\phi^{(\alpha)}} \phi\bar{\phi} + \min(\varphi_i, \varphi_j) K_{i\bar{\jmath}} \Phi^i \bar{\Phi}^{\bar{\jmath}} + \delta_{GS} M_P^2 \right) . \quad (23)
\end{aligned}
$$

In the simple case with only one singlet field ϕ, expanding the scalar potential in powers of ϕ we get

$$
V(\phi) = \widetilde{m}_\phi^2 |\phi|^2 + \frac{1}{2} D^2 , \quad (24)
$$

where $\widetilde{m}_\phi^2 = \left(1 + 3 n_\phi^{(\alpha)} \Theta_\alpha^2 \right) m_{3/2}^2$ is the F-term soft mass of ϕ induced by dilaton/moduli breaking. In (24) we used the fact that ϕ appears in the superpotential only through the Yukawa couplings to matter. The minimization of the potential (24) gives $g_X < D_X >= \prod_{\alpha=1}^{p} t_\alpha^{-n_\phi^{(\alpha)}} \widetilde{m}_\phi^2$. The D-breaking is induced by the soft mass $\widetilde{m}_\phi^2$ which is generated by the F-type dilaton/moduli breaking. The breaking of $U(1)_X$ yields a massive real scalar field of mass $\sqrt{2} g_X \varepsilon M_P$.

In the following, we place ourselves in the case where

$$
\partial_\alpha Z_{i\bar{\jmath}}, \partial_{\bar{\alpha}} Z_{i\bar{\jmath}}, \partial_\alpha Y_{ij} \simeq 0 , \quad (25)
$$

but still they have nontrivial associated modular weights. For the Yukawa couplings, this happens for example for functions of the form (keeping only an overall moduli field T) $Y \sim c + e^{-T}$ in the large radius (moduli) limit (see [12] for a more detailed discussion on this point). The Kähler off-diagonal terms $Z_{i\bar{\jmath}}$ are related to Yukawas through (15) and the explicit moduli dependence is probably closely related to that of the Yukawas.

From (18), (19) and (22), one obtains the expression for the soft scalar mass matrices as follows,

$$
\frac{\widetilde{m}_{i\bar{\jmath}}^2}{m_{3/2}^2} = \left(1 + \varphi_i + 3\Theta_\alpha^2 (n_i^{(\alpha)} + \varphi_i n_\phi^{(\alpha)}) \right) \delta_{ij} + \left[-\frac{1}{2} |\varphi_i - \varphi_j| + \frac{3}{2} (n_{i\bar{\jmath}}^{(\alpha)} + \bar{n}_{i\bar{\jmath}}^{(\alpha)}) \right.
$$

$$\times \Theta_\alpha^2 - \sqrt{3}\Theta_\alpha(n_{i\bar{j}}^{(\alpha)}\theta_{ji}(\varphi_j - \varphi_i) + \bar{n}_{i\bar{j}}^{(\alpha)}\theta_{ij}(\varphi_i - \varphi_j)) - 3\Theta_\alpha\Theta_\beta n_{i\bar{j}}^{(\alpha)}\bar{n}_{i\bar{j}}^{(\alpha)}\Big]\hat{Z}_{i\bar{j}}^\Phi \, , (26)$$

where $\hat{Z}_{i\bar{j}}^\Phi = \prod_\alpha t_\alpha^{\frac{1}{2}(n_{i\bar{j}}^{(\alpha)}+\bar{n}_{i\bar{j}}^{(\alpha)})}Z_{i\bar{j}}^\Phi\hat{\varepsilon}^{|\varphi_i-\varphi_j|}$ and $\theta_{ij} = \theta(\varphi_i - \varphi_j)$. This general result simplifies in our case of physical interest, i.e. with the mass hierarchies given solely by the $U(1)_X$ symmetry. Using eq. (16) we find that the combination $n_i^{(\alpha)} + \varphi_i n_\phi^{(\alpha)}$ is flavour blind and we get

$$\widetilde{m}_{i\bar{i}}^2 - \widetilde{m}_{j\bar{j}}^2 \quad = \quad (\varphi_i - \varphi_j)m_{3/2}^2 \qquad (27)$$

for $\Phi^i = Q^i, U^i, D^i, L^i, E^i$. Therefore the splitting in the diagonal elements of the sfermion masses is independent of the parameters Θ_α and proportional to the charge differences, which fix also the fermion masses. For example, in the Froggatt- Nielsen case ($\varphi_i \geq \varphi_j$ for $i \geq j$), we have $\frac{m_i^U}{m_j^U} \sim \hat{\varepsilon}^{q_{ij}+u_{ij}}$ and we get the fermion-sfermion mass predictions ($\widetilde{m}_{q_i}^2 \equiv \widetilde{m}_{q_{ii}}^2$, etc.)

$$m_{3/2}^2 \, ln\frac{m_i^U}{m_j^U} = (\widetilde{m}_{q_i}^2 - \widetilde{m}_{q_j}^2 + \widetilde{m}_{u_i}^2 - \widetilde{m}_{u_j}^2) \, ln\hat{\varepsilon} \qquad (28)$$

and similar relations for down-quarks and leptons. Also, using (27), we deduce that the higher the $U(1)_X$ charge (i.e. the smaller the corresponding Yukawa couplings) the larger the soft scalar mass. Hence, in that model the spectrum of the matter field superpartners has *inverted hierarchy* compared to that of the associated fermions.

Furthermore, combining (13) and (26) and introducing the tree-level gaugino masses $M = \sqrt{3}\sin\theta m_{3/2}$, we obtain the relations

$$\widetilde{m}_{q_i}^2 + \widetilde{m}_{u_j}^2 + \widetilde{m}_{h_2}^2 \quad = \quad M^2 + (q_i + u_j + h_2)m_{3/2}^2 \, . \qquad (29)$$

Similar relations are obtained for d-type squarks and sleptons. Notice that the combination of charges in the r.h.s. is precisely the power of $\hat{\epsilon}$ in the effective Yukawa couplings. So, since the top Yukawa is of $O(1)$, $q_3 + u_3 + h_2 = 0$.

Due to the non-diagonal form of the Kähler potential, the scalar mass matrices are not diagonal, in contrast to the usual computations in the literature. Moreover, under a stronger assumption $n_{\phi,i\bar{j}}^{(\alpha)} = \bar{n}_{\phi,i\bar{j}}^{(\alpha)} = 0$, we find that the flavour-dependent effects in the off-diagonal terms do not depend on the unknown parameters $\Theta_\alpha, n_i^{(\alpha)}$. Remarkably enough, in this case the contribution to $(\widetilde{m}_{i\bar{j}}^2)_F$ from supersymmetry breaking along the ϕ direction to (26) vanishes in the leading order in $< \phi > /M_P$.

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All these equations for scalar masses are to be understood at energies of the order M_P, and lead to low energy relations after renormalization.

The non-diagonal terms in K and W affect the trilinear soft terms V_{ijk}, too. The general expression for the trilinear terms corresponding to the fields with $< G^i >=< G_i >= 0$ is [20]

$$V_{ijk} = \left[(G^A D_A + 3) \frac{W_{ijk}}{W} \right] m_{3/2}^2 \ , \tag{30}$$

where D stands for the covariant derivative in the Kähler manifold. Here we give only the expressions for the most contrained case, i.e. where eqs.(16) and (25) are valid, with $n_{\phi,ij}^{(\alpha)} = 0$. Once again we work with canonical normalization of the scalar fields. With this convention and in the leading order of the small parameter $\hat{\varepsilon}$ the connections in the covariant derivatives in (30) take the simple form

$$G^\alpha \Gamma_{\alpha i}^j = \sqrt{3} n_i^{(\alpha)} \Theta_\alpha \delta_i^j + \frac{\sqrt{3}}{2} |\varphi_i - \varphi_j| n_\phi^{(\alpha)} \Theta_\alpha \hat{\varepsilon}^{|\varphi_i - \varphi_j|} Z_{i\bar{j}}^\Phi \ ,$$

$$G^\phi \Gamma_{\phi i}^j = (1 - \sqrt{3} n_\phi^{(\alpha)} \Theta_\alpha)(\varphi_i - \varphi_j)\theta(\varphi_i - \varphi_j)\hat{\varepsilon}^{|\varphi_i - \varphi_j|} Z_{i\bar{j}}^\Phi \ . \tag{31}$$

The final result for the triscalar coefficient V_{ia}^U, for example, reads (with $\hat{Y}^U = e^{K/2} Y^U$)

$$\frac{1}{m_{3/2}} V_{ia}^U = \left[-\sqrt{3} \sin\theta + (q_i + u_a + h_2) \right] \hat{Y}_{ia}^U \tag{32}$$

$$-\frac{1}{2} \left(\sum_j |q_i - q_j| Z_{i\bar{j}}^Q \hat{Y}_{ja} \hat{\varepsilon}^{|q_i - q_j|} + \sum_b |u_b - u_a| Z_{a\bar{b}}^U \hat{Y}_{ib} \hat{\varepsilon}^{|u_b - u_a|} \right)$$

and similar expressions hold for V^D and V^L with obvious replacements. Notice that the matrices $\hat{Y}$ have hierarchical entries and that the last line in (32) contain terms not directly proportional to the Yukawa coupling $\hat{Y}_{ia}$, but rotated in the flavour space. The last terms in (32) come from $G^\phi D_\phi$, namely, from supersymmetry breaking along the ϕ direction. It is useful to introduce the matrices

$$\frac{1}{m_{3/2}} (A_L)_{ij} = (q_i + \frac{h_2}{2})\delta_{ij} - \frac{1}{2} |q_i - q_j| Z_{i\bar{j}}^Q \hat{\varepsilon}^{|q_i - q_j|} \ , \tag{33}$$

$$\frac{1}{m_{3/2}} (A_R)_{ba} = (u_a + \frac{h_2}{2})\delta_{ab} - \frac{1}{2} |u_a - u_b| Z_{a\bar{b}}^U \hat{\varepsilon}^{|u_a - u_b|} \ .$$

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Then

$$V_{ia}^U = -M\hat{Y}^U + A_L\hat{Y}^U + \hat{Y}^U A_R \ . \tag{34}$$

This parametrization was already used in [21] in the context of the models proposed in [12]. The simplicity of the results follows from nontrivial cancellations between G^α and G^ϕ contributions due to eq.(16).

As a particular case, in the absence of the $U(1)_X$ symmetry we recover the minimal MSSM. Therefore, in this case, imposing appropriate modular transformations for the renormalizable Yukawa couplings and under the assumption that they do not depend explicitly on moduli fields, we get *family-universal* soft terms related by

$$\widetilde{m}_q^2 + \widetilde{m}_u^2 + \widetilde{m}_{h_2}^2 = \widetilde{m}_q^2 + \widetilde{m}_d^2 + \widetilde{m}_{h_1}^2 = \widetilde{m}_l^2 + \widetilde{m}_e^2 + \widetilde{m}_{h_1}^2 = M^2 \, ,$$
$$A^U = A^D = A^E = -M \, . \tag{35}$$

These simple relations appeared already in the litterature in different contexts [12, 22] and can be explained here by the modular invariance conditions of the Yukawa couplings combined with dilaton/moduli breaking. It is interesting to compare our results with those obtained in [22], where the role of the horizontal symmetry is played by modular symmetries. The essential difference is in the predictions for the soft terms which in [22] turn out to be flavour blind.

5 Mass terms in the Higgs sector

In this section we discuss the predictions for the mass parameters of the Higgs sector, the soft masses m_1^2, m_2^2, the μ parameter of MSSM and its associated soft breaking term $B\mu$.

In order to avoid the usual μ-problem of the MSSM, we assume here that both the μ and $B\mu$ terms are effective interactions resulting from the Kähler potential after supersymmetry breaking [23]. Actually, a H_1H_2 term in the superpotential is forbidden by the $U(1)_X$ symmetry if $h_1 + h_2 < 0$ (in the case of only one singlet field considered in this section). For $h_1 + h_2 = 0$, the absence of the corresponding mass term in low energy string models is equivalent (after decoupling of heavy modes) to the presence of massless Higgs doublets in the effective theory. Instead, for $h_1 + h_2 > 0$, an effective μ−term of $O(\epsilon^{h_1+h_2}M_P)$ would be possible, which is inconsistent with the proper breaking of the electroweak symmetry. This μ−problem could

be solved by further symmetries, e.g. modular symmetries (provided that they would allow for appropriate terms in the Kähler potential, of course). The allowed values of $(h_1 + h_2)$ are strongly correlated to the generation of fermion mass hierarchies in this approach with horizontal $U(1)_X$ symmetry and Green-Schwarz anomaly cancellation. One derives [1] - [5] the relation $(h_1 + h_2) \ln \epsilon \simeq Tr \ln(Y^D/Y^E)$, which, after substituting the physical fermion masses, favours the values $(h_1 + h_2) = 0, -1$, with the values $(+1)$ and (-2) marginally allowed.

For the purpose of generating the μ and $B\mu$ terms, we consider here two classes of Kähler potentials for the Higgs doublets. As a first instance, we just extend the general approach of sections 2 and 4 to include a $H_1 H_2$ term in the Kähler potential, with the $U(1)_X$ symmetry restored by powers of ϕ or $\bar{\phi}$, as follows:

$$
\begin{aligned}
K \;=\; & ...+ \prod_{\alpha=1}^{p} t_\alpha^{n_1^{(\alpha)}} |H_1|^2 + \prod_{\alpha=1}^{p} t_\alpha^{n_2^{(\alpha)}} |H_2|^2 \\
& + Z \left(\left(\frac{\phi}{M_P} \right)^{h_1+h_2} H_1 H_2 \, \theta(h_1 + h_2) + \right. \\
& \quad \prod_{\alpha=1}^{p} t_\alpha^{n_1^{(\alpha)}+n_2^{(\alpha)}} \left(\frac{\bar{\phi}}{M_P} \right)^{-(h_1+h_2)} H_1 H_2 \, \theta(-h_1 - h_2) \\
& \left. \hspace{6cm} +h.c. \right) ,
\end{aligned} \tag{36}
$$

where Z is of $O(1)$ and the dots stand for terms independent of H_1, H_2. Modular invariance of eq. (36) demands the following relation

$$
n_1^{(\alpha)} + n_2^{(\alpha)} + (h_1 + h_2) \, n_\phi^{(\alpha)} \;=\; 0 , \tag{37}
$$

if Z is assumed to be t_α−independent. In turn, imposing (37) forbids the presence of the μ-term directly in the superpotential, even in the case $h_1 + h_2 > 0$. From (36), on expects the μ-term to be of the order $O(\epsilon^{|h_1+h_2|} m_{3/2})$, also favouring small values of $(h_1 + h_2)$.

With the Kähler potential completed as in eqs. (36) and (37), we return to the question of the uniqueness of the solution (7) for the vanishing of the D-term of the $U(1)_X$ group. It reads

$$
D_X \;=\; g_X \left(-\frac{\partial K}{\partial \phi} \phi + \sum_{i=1,2} h_i \frac{\partial K}{\partial H_i} H_i + ... + \delta_{GS} M_P^2 \right) . \tag{38}
$$

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The vanishing of the $SU(3)\otimes SU(2)\otimes U(1)$ D-terms requires $\frac{\partial K}{\partial \ln H_1} = \frac{\partial K}{\partial \ln H_2}$, hence, for the canonically normalized Higgs fields, $H_1 = \pm H_2 = v$. In the absence of any relevant term in the superpotential, there are continuously degenerate solutions satisfying $\delta_{GS} M_P^2 = \phi^2 - (h_1 + h_2)v^2$. This degeneracy is removed by supersymmetry breaking assumed in the dilaton and moduli sector, which yields the scale $m_{3/2} \ll \epsilon M_P$ and the soft terms. The resulting scalar potential along the flat direction can be analysed through an expansion in powers of ϵ. At leading order, we get (for $h_1 + h_2 \neq 0$)

$$V = (2 + h_1 + h_2)m_{3/2}^2 v^2 + const. . \tag{39}$$

Therefore, for $(h_1 + h_2) \geq -2$, the minimum is for $\widetilde{\phi} = \delta_{GS}^{1/2} M_P$, $v = 0$ and $D_X = \widetilde{m}_\phi^2/g_X$. The same conclusion holds for $h_1 + h_2 = 0$, as will be evident from the discussion below eq. (45). For $(h_1 + h_2) = -2$, the continuous degeneracy persists at this level of approximation. For $(h_1 + h_2) < -2$, the minimum of V is for $\phi = 0$ and $(h_1 + h_2)v^2 = \delta_{GS} M_P^2$, which is physically uninteresting.

We proceed to calculate the effective lagrangian in the Higgs doublet sector. The scalar terms are obtained from (22) and the μ and $B\mu$ effective parameters are derived from the general supergravity expressions for a fermion supersymmetric mass term $(M_{1/2})_{ij}$ and analytic scalar soft mass $(M_0^2)_{ij}$ (i, j are matter indices) [20]

$$
\begin{aligned}
(M_{1/2})_{ij} &= m_{3/2}\left(D_i G_j + \frac{1}{3}G_i G_j\right) , \\
(M_0^2)_{ij} &= m_{3/2}^2\left(G^A D_A + 2\right) D_i G_j .
\end{aligned} \tag{40}
$$

We find the following results:

$$
\begin{aligned}
\widetilde{m}_i^2 &= \left(1 + h_i + 3\Theta_\alpha^2\left(n_i^{(\alpha)} + h_i n_\phi^{(\alpha)}\right)\right)m_{3/2}^2 , \\
\mu &= m_{3/2}\left(1 + (h_1 + h_2)\theta(-h_1 - h_2)\right)Z\epsilon^{|h_1+h_2|} , \\
B &= m_{3/2}\left(2 + (h_1 + h_2)\theta(h_1 + h_2)\right) .
\end{aligned} \tag{41}
$$

The simple prediction $B = 2$ for $(h_1 + h_2) \leq 0$ arises from a cancellation between the geometric and D_X-term contributions to the $B\mu$ analytic coupling, due to the relation (37). The latter is absent for $h_1 + h_2 > 0$ because of the analyticity of the coupling $H_1 H_2 (\phi/M_P)^{h_1+h_2}$. Though the parameters $\widetilde{m}_1^2$ and $\widetilde{m}_2^2$ depend on the unknown quantities Θ_α^2, their sum does not

$$\widetilde{m}_1^2 + \widetilde{m}_2^2 = (2 + h_1 + h_2)m_{3/2}^2 . \tag{42}$$

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In order to decide if these predictions are consistent with the requirements of $SU(2) \otimes U(1)$ breaking, one has to renormalize the parameters down to the Fermi scale. In our models the relevant parameters take a very simple form. As discussed in section 3, $Y_{33}^U \sim O(1)$ implies $h_2 + q_3 + u_3 = 0$, which, in turn, gives

$$A_t = -M = -\sqrt{3}m_{3/2}\sin\theta ,$$
$$\widetilde{m}_{h_2}^2 + \widetilde{m}_{U_3}^2 + \widetilde{m}_{Q_3}^2 = M^2 . \tag{43}$$

Therefore the low energy parameters in the Higgs sector depend only on their initial values (41), on the top mass, relatively well-known, and on the gaugino masses parameter, M. One gets the following approximate results:

$$(\widetilde{m}_1^2 + \widetilde{m}_2^2)_{|M_Z} = (\widetilde{m}_1^2 + \widetilde{m}_2^2) - \frac{1}{2}(10\rho - 2 - \rho^2)M^2 ,$$
$$B_{|M_Z} = B - \frac{1-\rho}{2}M ,$$
$$\mu^2_{|M_Z} = 2(1-\rho)^{1/2}\mu^2 , \tag{44}$$

where $\rho = m_t^2 / m_{t\,crit}^2$ is the ratio between the physical and the infrared fixed-point values of the (running) top mass squared. Let us consider the following necessary conditions for $SU(2) \otimes U(1)$ breaking in the MSSM model:

$$2|B\mu|_{|M_Z} \leq (\widetilde{m}_1^2 + \widetilde{m}_2^2 + 2\mu^2)_{|M_Z} ,$$
$$B^2\mu^2_{|M_Z} \geq (\widetilde{m}_1^2 + \mu^2)(\widetilde{m}_2^2 + \mu^2)_{|M_Z} . \tag{45}$$

For $(h_1 + h_2) = -1$, $\mu = 0$ indicating that, in this case, the gauged symmetry $U(1)_X$ implies an accidental $U(1)_{PQ}$ symmetry. It is well-known that this symmetry is inconsistent with phenomenology.

For $(h_1 + h_2) = 0$, one has $\widetilde{m}_1^2 + \widetilde{m}_2^2 + 2\mu^2 - 2B\mu = 2(1 - Z)^2 m_{3/2}^2$ at the unification scale. This implies (for $Z \neq 1$) that the scalar potential at this scale has a minimum for $H_1 = \pm H_2 = v = 0$. The relations (44) are satisfied if $|M| < (1 - Z)m_{3/2}$. In this case we predict low values for $\tan\beta$.

For $(h_1 + h_2) = 1$, $B = 3m_{3/2}$, $\mu = Z\epsilon m_{3/2}$, $\widetilde{m}_1^2 + \widetilde{m}_2^2 = 3m_{3/2}^2$. The breaking of $SU(2) \otimes U(1)$ then needs a fine-tuning between M and $m_{3/2}$ (to $O(\epsilon^2)$), with $\tan\beta \sim O(1/\epsilon^2)$.

The case $(h_1 + h_2) = -2$, already plagued by vacuum degeneracy at high energies, leads to $B = 2m_{3/2}$, $\mu \sim \epsilon^2 m_{3/2}$ and $\widetilde{m}_1^2 + \widetilde{m}_2^2 = 0$. These values are inconsistent with (45) even after renormalization.

Summarizing, in this class of models, only those with $(h_1 + h_2) = 0$ seem to stand up to $SU(2) \otimes U(1)$ breaking requirements, if $M < (1 - Z)m_{3/2}$. As discussed in section 7, in this case the gaugino mass can help to suppress FCNC effects. Radiative corrections will affect to some extent the contraints (45), but a detailed phenomenological discussion of the Higgs sector is beyond the scope of this paper.

We now turn to a model which have been found in $(2, 2)$ superstring compactification [24]. The Kähler potential of such models is

$$K = -\ln\left[(T + \bar{T})(U + \bar{U}) - (H_1 + \bar{H}_2)(\bar{H}_1 + H_2)\right] + \dots , \qquad (46)$$

where T, U are two moduli fields and the dots stand for terms independent of H_1, H_2. The $U(1)_X$ symmetry of (46) demands $(h_1 + h_2) = 0$. The modular transformations associated to T and U now read

$$
\begin{aligned}
T & \rightarrow \frac{aT - ib}{icT + d} , \\
H_{1,2} & \rightarrow \frac{1}{icT + d} H_{1,2} , \\
U & \rightarrow U - \frac{ic}{icT + d} H_1 H_2 .
\end{aligned}
\qquad (47)
$$

The soft terms are calculated to be

$$
\begin{aligned}
\widetilde{m}_{h_i}^2 & = \left(1 + h_i - 3(\Theta_T^2 + \Theta_U^2) + 3h_i(n_\phi^{(T)}\Theta_T^2 + n_\phi^{(U)}\Theta_U^2)\right) m_{3/2}^2 , \\
\mu & = \left(1 + \sqrt{3}\,(\Theta_T + \Theta_U)\right) m_{3/2} , \\
B\mu & = \left(2 + 2\sqrt{3}\,(\Theta_T + \Theta_U) + 6\Theta_T\Theta_U\right) m_{3/2}^2 .
\end{aligned}
\qquad (48)
$$

This case is much less predictive than the previous one (the soft terms depend on the unknown parameters $\Theta_{T,U}$), mainly because of the lack of a relation as (37) between the moduli weights and the $U(1)_X$ charges . As noticed in [12] and [25] , the inequality (45) is saturated at the classical level, corresponding to a flat direction in the classical potential, even if the degeneracy between $\widetilde{m}_1^2$ and $\widetilde{m}_2^2$ is removed by the D_X−terms. From (44) one deduces that in order to fulfill the first condition in (45), one needs $M/m_{3/2} \sim O(\frac{1}{10})$, namely, a moduli dominated supersymmetry breaking. Furthermore, from the second condition in (45) and the requirement of the proper value for M_Z we get $\widetilde{m}_2^2 \sim O(-M_Z^2)$ and low $\tan\beta$ values. It follows from eq.(43) that $\widetilde{m}_{Q_3}, \widetilde{m}_{U_3} \sim O(M_Z)$. Consequently, in this case we expect light gauginos and light third generation scalars.

It is possible to arbitrarily extend the model to $(h_1+h_2) > 0$ by replacing in (46) and (47), $H_1 H_2 \rightarrow \phi^{h_1+h_2} H_1 H_2$. But there is no theoretical basis for such models anymore.

6 $U(1) \otimes U'(1)$ horizontal symmetry

It has been demonstrated in ref. [10] that, in models with two $U(1) \otimes U'(1)$ symmetries, further suppression of the FCNC effects is possible.

Hence, we now extend our study to this class of models. We introduce two SM singlet fields ϕ_1 and ϕ_2. Their charges can be chosen to be $\phi_1 (-1,0)$ and $\phi_2 (0,-1)$ (except for the case of proportional charges). Then, no superpotential term $W(\phi_1, \phi_2)$ can be written for zero vev's of matter fields; the vev's of the fields ϕ_1, ϕ_2 are fixed by the Fayet-Iliopoulos term. The superpotential W and the Kähler potential K (with $\varepsilon_1 = \phi_1/M_P$ and $\varepsilon_2 = \phi_2/M_P$) are:

$$
\begin{aligned}
W &= \sum_{ij} \left[Y_{ij}^U \, \theta_{q_i,u_j,h_2} \, \theta_{q_i',u_j',h_2'} \, \varepsilon_1^{q_i+u_j+h_2} \varepsilon_2^{q_i'+u_j'+h_2'} Q^i U^j H_2 + \right. \\
&\qquad Y_{ij}^D \, \theta_{q_i,d_j,h_1} \, \theta_{q_i',d_j',h_1'} \, \varepsilon_1^{q_i+d_j+h_1} \varepsilon_2^{q_i'+d_j'+h_1'} Q^i D^i H_1 + \\
&\qquad \left. Y_{ij}^E \, \theta_{\ell_i,e_j,h_1} \, \theta_{\ell_i',e_j',h_1'} \, \varepsilon_1^{\ell_i+e_j+h_1} \varepsilon_2^{\ell_i'+e_j'+h_1'} L^i E^j H_1 \right],
\end{aligned}
$$

$$
\begin{aligned}
K &= K_0 \left(T_\alpha, \bar{T}^{\bar{\alpha}}\right) - \ln\left(S + \bar{S}\right) + \prod_{\alpha=1}^{p} t_\alpha^{n_{\phi_1}^{(\alpha)}} \phi_1 \bar{\phi}_1 + \prod_{\alpha=1}^{p} t_\alpha^{n_{\phi_2}^{(\alpha)}} \phi_2 \bar{\phi}_2 \\
&\quad + \sum_{\Phi^i = Q^i, U^i, D^i, L^i, E^i} K_{i\bar{j}}^{\Phi} \Phi^i \bar{\Phi}^{\bar{j}},
\end{aligned}
$$

$$
\begin{aligned}
K_{i\bar{j}}^{\Phi} &= \delta_{i\bar{j}} \prod_{\alpha=1}^{p} t_\alpha^{n_i^{(\alpha)}} + Z_{i\bar{j}}^{\Phi} \left[\theta_{ij} \theta_{ij}' \prod_{\alpha=1}^{p} t_\alpha^{n_j^{(\alpha)}} \varepsilon_1^{\varphi_i - \varphi_j} \varepsilon_2^{\varphi_i' - \varphi_j'} \right. \\
&\qquad + \theta_{ji} \theta_{ij}' \prod_{\alpha=1}^{p} t_\alpha^{n_j^{(\alpha)} + n_{\phi_1}^{(\alpha)}(\varphi_j - \varphi_i)} \bar{\varepsilon}_1^{\varphi_j - \varphi_i} \varepsilon_2^{\varphi_i' - \varphi_j'} \\
&\qquad + \theta_{ij} \theta_{ji}' \prod_{\alpha=1}^{p} t_\alpha^{n_i^{(\alpha)} + n_{\phi_1}^{(\alpha)}(\varphi_i - \varphi_j)} \varepsilon_1^{\varphi_i - \varphi_j} \bar{\varepsilon}_2^{\varphi_j' - \varphi_i'} \\
&\qquad \left. + \theta_{ji} \theta_{ji}' \prod_{\alpha=1}^{p} t_\alpha^{n_i^{(\alpha)}} \bar{\varepsilon}_1^{\varphi_j - \varphi_i} \bar{\varepsilon}_2^{\varphi_j' - \varphi_i'} \right] + \dots, \quad (49)
\end{aligned}
$$

where the previous notation is used and we define $\theta_{(a,b,c)} = \theta(a + b + c)$,

$\theta_{ij} = \theta(\varphi_i - \varphi_j)$ and $\theta'_{ij} = \theta(\varphi'_i - \varphi'_j)$.

The $D-$term contributions to the scalar potential are,

$$V_D \;=\; \frac{g_1^2}{2}\left(K_i\varphi_i\Phi^i - K_{\phi_1}\phi_1 + \xi_1\right)^2 + \frac{g_2^2}{2}\left(K_i\varphi'_i\Phi^i - K_{\phi_2}\phi_2 + \xi_2\right)^2 ,$$

where ξ_1, ξ_2 are the Fayet-Iliopoulos terms. We can, of course, always define one linear combination of the two $U(1)$'s which is anomaly free, but we prefer to work in a basis where the ϕ_i charges are simple. Only the simplest and most predictive case $n^{(\alpha)}_{\Phi,ij} = n^{(\alpha)}_{\Phi,i\bar{j}} = \bar{n}^{(\alpha)}_{\Phi,i\bar{j}} = 0$ for $\Phi = Q$, U, D, E, L is considered here.

In analogy with the case of one $U(1)$ symmetry, the relation betwen horizontal charges and modular weights reads:

$$n^{(\alpha)}_j - n^{(\alpha)}_i \;=\; n^{(\alpha)}_{\phi_1}(\varphi_i - \varphi_j) + n^{(\alpha)}_{\phi_2}(\varphi'_i - \varphi'_j). \qquad (50)$$

The relations (50) enable us to rewrite the matter field metric as:

$$K^\Phi_{i\bar{j}} \;=\; \prod_{\alpha=1}^{p} t_\alpha^{(n^{(\alpha)}_i + n^{(\alpha)}_j)/2}\left[\delta_{ij} + \hat{Z}^\Phi_{i\bar{j}}\right] ,$$

where

$$\hat{Z}^\Phi_{i\bar{j}} \;=\; Z^\Phi_{i\bar{j}} \prod_{\alpha=1}^{p} t_\alpha^{\frac{1}{2}(n^{(\alpha)}_{\phi_1}|\varphi_i-\varphi_j| + n^{(\alpha)}_{\phi_2}|\varphi'_i-\varphi'_j|)}$$
$$\left[\theta_{ij}\,\theta'_{ij}\varepsilon_1^{\varphi_i-\varphi_j}\varepsilon_2^{\varphi'_i-\varphi'_j} + \theta_{ji}\,\theta'_{ij}\bar{\varepsilon}_1^{\varphi_j-\varphi_i}\varepsilon_2^{\varphi'_i-\varphi'_j} + \right.$$
$$\left. \theta_{ij}\,\theta'_{ji}\varepsilon_1^{\varphi_i-\varphi_j}\bar{\varepsilon}_2^{\varphi'_j-\varphi'_i} + \theta_{ji}\,\theta'_{ji}\bar{\varepsilon}_1^{\varphi_j-\varphi_i}\bar{\varepsilon}_2^{\varphi'_j-\varphi'_i} \right] .$$

The soft scalar mass matrices are as follows:

$$\widetilde{m}^2_{i\bar{j}} \;=\; \left(G_{i\bar{j}} - G^A \hat{R}_{i\bar{j}A\bar{B}} G^{\bar{B}}\right) m^2_{3/2} +$$
$$g_1\min(\varphi_i,\varphi_j)G_{i\bar{j}} < D_1 > + g_2\min(\varphi'_i,\varphi'_j)G_{i\bar{j}} < D_2 > , \quad (51)$$

where A, $B = \alpha$, ϕ_1, ϕ_2. In absence of the term $W(\phi_1,\phi_2)$ in the superpotential, one finds by minimization of the scalar potential

$$g_i \prod_{\alpha=1}^{p} t_\alpha^{n^{(\alpha)}_{\phi_i}} < D_i >= \widetilde{m}^2_{\phi_i}, \text{ where } \widetilde{m}^2_{\phi_i} = (1 + 3n^{(\alpha)}_{\phi_i}\Theta^2_\alpha)m^2_{3/2}.$$

The final expression for the scalar masses in the canonical basis reads:

$$
\begin{aligned}
\widetilde{m}^2_{i\bar{\jmath}} \;=\;& m^2_{3/2}\Big(\ \big(1+\varphi_i+\varphi'_i+3\Theta^2_\alpha(n^{(\alpha)}_i+\varphi_i n^{(\alpha)}_{\phi_1}+\varphi'_i n^{(\alpha)}_{\phi_2})\big)\delta_{i\jmath} \\
& -\frac{1}{2}\Big(|\varphi_i-\varphi_\jmath|+|\varphi'_i-\varphi'_\jmath|\Big)\hat{Z}^\Phi_{i\bar{\jmath}} \\
& -(\varphi_i-\varphi_\jmath)(\varphi'_\jmath-\varphi'_i)\theta_{i\jmath}\theta'_{\jmath i}\hat{Z}^\Phi_{i\bar{\jmath}}-(\varphi_\jmath-\varphi_i)(\varphi'_i-\varphi'_\jmath)\theta_{\jmath i}\theta'_{i\jmath}\hat{Z}^\Phi_{i\bar{\jmath}}\ \Big)
\end{aligned}
\tag{52}
$$

As for the one $U(1)_X$ symmetry case, the only dependence on the unknown parameters Θ_α, $n^{(\alpha)}_i$ is contained in a diagonal flavour independent term. This is a consequence of the eq. (50) which leads to nontrivial cancellations between F-terms and D-terms. Now, however, supersymmetry breaking along directions G^{ϕ_i} do contribute to the soft masses; in eq. (52) the last two lines come from $G^{\phi_1}G^{\bar{\phi}_2}\frac{\partial K_{i\bar{\jmath}}}{\partial\phi_1\partial\phi_2}+h.c.$

For the trilinear coefficient $\hat{V}^U_{ia}$, we obtain, by using (50):

$$
\begin{aligned}
V^U_{ia} \;=\;& m_{3/2}\Big(\ \big(-\sqrt{3}\sin\theta+(q_i+u_a+h_2)+(q'_i+u'_a+h'_2)\big)\hat{Y}^U_{ia} \\
& -\frac{1}{2}(|q_i-q_k|+|q'_i-q'_k|)\hat{Z}^\Phi_{i\bar{k}}\hat{Y}^U_{ka} \\
& -\frac{1}{2}(|u_a-u_c|+|u'_a-u'_c|)\hat{Z}^\Phi_{a\bar{c}}\hat{Y}^U_{ic}\ \Big)\ ,
\end{aligned}
\tag{53}
$$

where we have defined $\hat{Y}^U=e^{K/2}Y^U$, as in section 4. It is straightforward to check that, using the results (52) and (53), we get predictions similar to eqs.(27) and (29) with $\varphi_i\to\varphi_i+\varphi'_i$.

An important consequence of eqs.(52) and (53) is that the flavour off-diagonal terms can vanish only if $\varphi_i=\varphi_j$ and $\varphi'_i=\varphi'_j$. In this case, also the diagonal terms are flavour independent. However, the corresponding fermion mass matrix will not have the required hierarchical structure. Thus, it is impossible to have the sfermion mass matrices *diagonal and degenerate* and simultaneously to keep the hierarchical structure of the corresponding fermion mass matrices. This result can be generalized to an arbitrary number of abelian symmetries. It is, nevertheless, still possible to have degeneracy between some *diagonal entries* in sfermion mass matrices $\widetilde{m}^2_i=\widetilde{m}^2_j$ by choosing models with $\varphi_i+\varphi'_i=\varphi_j+\varphi'_j$. We shall return to this discussion in the next section.

7 Phenomenological aspects

We have proposed a class of supergravity models with horizontal abelian gauge symmetries and modular invariance in which the hierarchies in the fermion mass spectrum are entirely due to the $U(1)$ symmetries. They have several interesting phenomenological aspects which can be grouped as very general qualitative features and more model dependent results. On the general side, the most important are :

i) high predictivity for the supersymmetric spectrum; the sfermion spectrum and the Higgs boson spectrum is strongly correlated with the fermion masses and mixing angles (in the simplest case of one $U(1)$ it is entirely determined) and shows the family dependence inverse to fermions (lighter sfermions correspond to heavier fermions).

ii) generic presence of flavour mixing effects already at the Planck scale in the squark mass soft terms and trilinear terms; again, these effects are strongly correlated with the pattern of fermion masses and not only at the order of magnitude level (like in models with $U(1)$ symmetries alone) but with the relative magnitude of different terms fixed by the horizontal charges.

iii) qualitative consistency with the present experimental contraints, which is remarkable in view of the rigidity of the models; in particular one can construct models which give FCNC effects at low energy suppressed strongly enough to meet the experimental limits. However, at the same time our class of models typically gives FCNC effects which are stronger than expected from the universality ansatz at the Planck scale and with predictible dependence on the up-down, left-right sectors. Thus, this class of models is suggestive of very rich future phenomenology in the domain of FCNC effects, once the experimental sensitivity is improved. One should stress that we expect FCNC effects to be only little below the present limits.

On the more model dependent side, we can distinguish the two cases of one $U(1)$ and two $U(1)$ symmetries. With one $U(1)$ our results are particularly definite. The only acceptable $U(1)$ charge assignement for the Higgs fields is $h_1 + h_2 = 0$. The FCNC effects are predicted to be large, although still marginally acceptable for certain charge assignments. They have been discussed in some detail in ref. [11], [26]) and we do not repeat this discussion here. One should stress that with the moduli-dominated supersymmetry breaking, i.e. with small strong interaction renormalization effects in the RG running from M_P to M_Z (see section 5) the FCNC effects are indeed at the border line of the present experimental limits [27] for

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$(\delta_{MM}^{u,d})_{1,2} = \frac{(\Delta M^{u,d})_{1,2}^2}{M_{av}^2}$ $(M = L, R)$ and $\delta_{1,2}^{u,d} = \sqrt{(\delta_{LL}^{u,d})_{1,2}(\delta_{RR}^{u,d})_{1,2}}$ (defined in the quark mass diagonal basis; M_{av} is an averaged squark mass).

It was shown in [10] that the constraints from $K^0 - \bar{K}^0$ mixing can be satisfied in models with two $U(1)$ symmetries and two mass scales. Models with two $U(1)$ symmetries give, of course, more freedom in the $U(1)$ charge assignements consistent with the pattern of Yukawa matrices and, in consequence, ease the problem of FCNC effects.

One can identify certain qualitative features of such models. One way to suppress FCNC is to impose some partial degeneracy for the diagonal entries in the squark masses. One can see on general grounds that it is the diagonal non-degeneracy in the $U(1)$ basis which is the main source of FCNC effects. Let us suppose that the Cabibbo angle λ is obtained by diagonalizing Y^U (Y^D). Then in the quark physical basis $(\delta_{LL}^u)_{12}\left((\delta_{LL}^d)_{12}\right) \sim$ $\max(\frac{\widetilde{m}_1^2 - \widetilde{m}_2^2}{M_{av}^2}V_{us}, \frac{(\widetilde{m}^Q)_{12}^2}{M_{av}^2}) \simeq V_{us}$. This prediction seems to be valid for any model based on an arbitrary number of abelian horizontal symmetries. Too large a $D^0 - \bar{D}^0$ mixing is predicted (the $K^0 - \bar{K}^0$ mixing can be suppressed as in [10] or as in our example below). In our class of models, this result can be avoided by a proper choice of charges. Indeed, with $q_1 + q_1' = q_2 + q_2'$ one has $\widetilde{m}_1^2 = \widetilde{m}_2^2$ and the problem disappears.

We need to check if this charge assignment is consistent with Yukawa matrices. Writing $\frac{Y_{ia}^U}{Y_{33}^U} \sim \varepsilon_1^{q_{13}+u_{a3}}\varepsilon_2^{q_{13}'+u_{a3}'}\Theta(q_i + u_a + h_2)\Theta(q_i' + u_a' + h_2')$, etc. one can check that, after imposing $q_1 + q_1' = q_2 + q_2'$, we get $V_{us} \sim \left(\frac{\varepsilon_1}{\varepsilon_2}\right)^{q_{12}}$. So, we necessarily need two mass scales. It is easy to construct an acceptable explicit model. We choose $\varepsilon_1 = \lambda$, $\varepsilon_2 = \lambda^2$ and the charge differences

$$q_{13} = 1, q_{23} = 2, u_{13} = 5, u_{23} = 2, d_{13} = 3, d_{23} = 0,$$
$$q_{13}' = 1, q_{23}' = 0, u_{13}' = 0, u_{23}' = 0, d_{13}' = -1, d_{23}' = 0. \tag{54}$$

The quark mass matrices are then of the form

$$Y^U \sim \begin{pmatrix} \lambda^8 & \lambda^5 & \lambda^3 \\ \lambda^7 & \lambda^4 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix}, Y^D \sim \begin{pmatrix} \lambda^4 & \lambda^3 & \lambda^3 \\ 0 & \lambda^2 & \lambda^2 \\ 0 & 1 & 1 \end{pmatrix}.$$

For the squark masses we get

$$(\widetilde{m}_{LL})^2 \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^3 & \lambda^3 \\ \lambda^3 & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

$$(\widetilde{m}_{RR}^u)^2 \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^3 & \lambda^5 \\ \lambda^3 & 1 & \lambda^2 \\ \lambda^5 & \lambda^2 & 1 \end{pmatrix} , (\widetilde{m}_{RR}^d)^2 \sim \widetilde{m}^2 \begin{pmatrix} 1 & \lambda^5 & \lambda^3 \\ \lambda^5 & 1 & 0 \\ \lambda^3 & 0 & 1 \end{pmatrix}.$$

From the assignement $q_1 + q_1' = q_2 + q_2'$ we get $(\widetilde{m}_{LL})_{11}^2 = (\widetilde{m}_{LL})_{22}^2$. The charge assignement (54) gives also $d_2 = d_3$ and $d_2' = d_3'$. Therefore, $(\widetilde{m}_{RR}^d)_{22}^2 = (\widetilde{m}_{RR}^d)_{33}^2$ and $(\widetilde{m}_{RR}^d)_{23}^2 = (\widetilde{m}_{RR}^d)_{32}^2$. The most important FCNC effects remain in $(\delta_{LL}^{u,d})_{12} \sim \lambda^3, (\delta_{RR}^u)_{12} \sim \lambda^3, (\delta_{RR}^d)_{12} \sim \lambda^5$ and $\delta_{12}^d \sim \lambda^4$. These signals are below the present experimental bounds but can be tested in the next generation of experiments.

Finally, we address the question of new supersymmetric phases which may be dangerous for CP violations. On general grounds, the soft mass matrices for left and right handed squarks are hermitian, i.e. the diagonal terms are real. In the flavour off-diagonal terms (in the quark mass diagonal basis) arbitrary phases can be present. However, in any model with proper suppression of the FCNC effects those off-diagonal terms are suppressed. Thus, in the case of squark masses the problem of new phases is automatically solved together with the FCNC problem.

For the trilinear terms (L-R mixing terms in the complete squark mass matrices) the situation is not so simple. It is still true that the phases in the flavour off-diagonal terms are typically not dangerous, for the same reason as in case of soft masses. For instance, for one $U(1)$ and in the quark mass diagonal basis we have

$$\frac{V^U}{m_{3/2}} = -\frac{M}{m_{3/2}} Y_d^U + (U_L A_L U_L^\dagger) Y_d^U + Y_d^U (U_R A_R U_R^\dagger) \tag{55}$$

and similar expressions for V^D and V^E. In (55), U_L and U_R are unitary matrices which diagonalize the mass matrix $Y_d^U = U_L Y^U U_R^+$. The result can be expressed in the form $(i \neq j)$

$$\frac{v_2}{m_{3/2}} (V^U)_{ij} = A_{ij} \hat{\varepsilon}^{|q_{ij}|} m_j^U + B_{ij} \hat{\varepsilon}^{|u_{ij}|} m_i^U + \cdots , \tag{56}$$

where $m_i^U = (m_u, m_c, m_t)$. The $O(1)$ matrices A_{ij} and B_{ij} are easily computed; for example, in the Froggatt-Nielsen hierarchical case, using the notation $(U_L)_{ij} = c_i \delta_{ij} + d_{ij} \hat{\varepsilon}^{|q_{ij}|}$, we find $A_{ij} = (q_i - q_j) d_{ji}^* c_j - \frac{1}{2} |q_{ij}| c_i c_j^* Z_{ij}^\Phi$. The dots in (56) denote higher order terms.

However, there is no general principle to protect diagonal A-terms against new phases. They can be dangerous for the electric dipole moment of the

neutron. Actually, the relevant phases are $Im(A_{ii}M^*)$, with A_{ii} defined in eq. (2). Consequently, we get $Im(A_{ii}^U M^*) = (q_i + u_i + h_2)Im(m_{3/2}M^*)$ and all other possible phases in G^α are irrelevant here.

8 Conclusions

The main purpose of this paper is to propose a theory of flavour where the supersymmetric spectrum is completely determined and experimentally testable. We study effective superstring models with abelian horizontal gauge symmetries and modular invariances. It is shown that the horizontal charges and the modular weights have to be correlated if the hierarchy of fermion masses follows solely from the $U(1)$ symmetries. In consequence, the soft terms in the supersymmetric spectrum, including the Higgs boson mass terms, are determined by the quark masses and mixing angles. This results in a predictive framework for the scalar masses. Indeed, the splittings between the diagonal as well as the non diagonal entries in the squark and slepton soft mass matrices turn out to be independent of the direction of supersymmetry breaking and of the modular weights associated to the matter fields.

We consider models with one and two $U(1)$ symmetries. The latter have more freedom in the assignement of the $U(1)$ charges and allow for stronger suppression of FCNC effects. However, as a generic feature, models of our type give FCNC effects only little below the present experimental limits, and are suggestive of very rich future phenomenology in this domain.

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